Math, Functions and graphs (Tomáš Nečas)

Time

45 min

Toppic

Drawing graphs of linear, quadratic, exponential and goniometric functions from their formulas.

Lesson goals

The main goal is to train creating graphs.

Whole task first

The students are asked to solve some complex problem. They can choose from different difficulities.

Part one: introduction

The following examples are based on the art workshop of famous Professor Ypsilon, whose art arises from the belief that only mathematics can provide an adequate picture of reality. It might seem, that this paintings are made up of a set of mathematical characters and symbols, and no artistic feeling is present in them. But a well trained mathematitian puts the 0xy of the cartesian coordinate system (the axes are parallel to the image frame) in the plane of the image and assigns the elements on the screen to the elements of the set created by the artist. And that's a task for you!
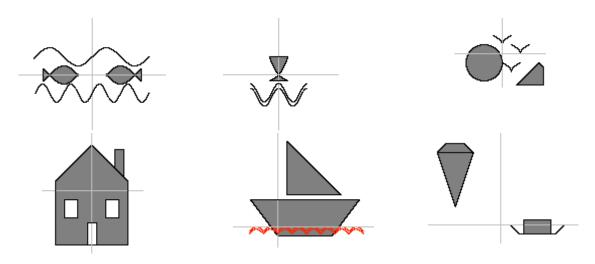
Part two: set of paintings of prof. Ypsilon

1.1

$$M_1 = \left\{ [x; y] \in R^2; \frac{\pi}{2} \le |x| \le \frac{7}{4}\pi \wedge |y| \le |\cos x| \right\}$$

$$M_2 = \left\{ [x; y] \in R^2; |x| \le 2\pi \wedge y = 2 + \sin x \right\}$$

$$M_3 = \left\{ [x; y] \in R^2; |x| \le 2\pi \wedge y = -2 + \sin 2x \right\}$$

1.2

$$M_1 = \left\{ [x; y] \in R^2; |x| \le 1 \wedge 2 \ge y \ge 3^{|x|} - 1 \right\}$$

$$M_2 = \left\{ [x; y] \in R^2; |x| \le 1 \wedge 3^{|-x|} - 1 \ge y \ge -\frac{2}{3} \right\}$$

$$M_3 = \left\{ [x; y] \in R^2; |x| \le \pi \wedge y = -2 + \cos 2x \right\}$$

$$M_4 = \left\{ [x; y] \in R^2; |x| \le \pi \wedge y = -\frac{5}{2} + \cos 2x \right\}$$

1.3

$$M_1 = \left\{ [x; y] \in R^2; (x+2)^2 + (y+1)^2 \le 4 \right\}$$

$$M_2 = \left\{ [x; y] \in R^2; |x| \le 1 \wedge y = 1 + \sqrt{|x|} \right\}$$

$$M_3 = \left\{ [x; y] \in R^2; |x-1| \le 1 \wedge y = -2 + \sqrt{|x-1|} \right\}$$

$$M_4 = \left\{ [x; y] \in R^2; |x-2| \le 1 \wedge y = \sqrt{|x-2|} \right\}$$

$$M_5 = \left\{ [x; y] \in R^2; \frac{3}{2} \le x \le \frac{9}{2} \wedge y \le -1 - |x-4| \right\}$$

1.4

$$M_1 = \left\{ [x; y] \in R^2; x \in \langle -4; 4 \rangle, -6 \le y \le -|x| + 5 \right\}$$

$$M_2 = \left\{ [x; y] \in R^2; x \in \langle 2,5; 3,5 \rangle, -x+5 \le y \le 4,5 \right\}$$

red:

$$M_3 = \left\{ [x; y] \in R^2; x \in \langle -3; -1,5 \rangle, y \in \langle -3; -1 \rangle \right\}$$

$$M_4 = \left\{ [x; y] \in R^2; x \in \langle 1,5; 3 \rangle, y \in \langle -3; -1 \rangle \right\}$$

$$M_5 = \left\{ [x; y] \in R^2; x \in \langle -0,5; 0,5 \rangle, y \in \langle -6; -3,5 \rangle \right\}$$

1.5

$$M_1 = \left\{ [x; y] \in R^2; x \in \langle -3; 9 \rangle, \frac{2}{3}|x| + \frac{2}{3}|x-6| - 5 \le y \le 3 \right\}$$

$$M_2 = \left\{ [x; y] \in R^2; x \in \langle 1; 7 \rangle, 3,5 \le y \le -x+10,5 \right\}$$

red:

$$M_3 = \left\{ [x; y] \in R^2; -\pi \le x \le 3\pi, y = 0,5|\sin 2x| - 0,5 \right\}$$

$$M_4 = \left\{ [x; y] \in R^2; -\pi \le x \le 3\pi, y = 0,5|\sin 2x| - 0,75 \right\}$$

1.6

$$M_1 = \left\{ [x; y] \in R^2; x \in \langle 1; 7 \rangle, y = \frac{1}{2}|2-x| + \frac{1}{2}|x-6| - 3 \right\}$$

$$M_2 = \left\{ [x; y] \in R^2; x \in \langle 2,5; 5,5 \rangle, y \in \langle -1; 0,5 \rangle \right\}$$

$$M_3 = \left\{ [x; y] \in R^2; x \in \langle -7; -3 \rangle, 3|x+5| + 2 \le y \le 8 \right\}$$

$$M_4 = \left\{ [x; y] \in R^2; x \in \langle -7; -3 \rangle, \right.$$

$$\left. 8 \le y \le -\frac{1}{2}|x+6| - \frac{1}{2}|x+4| + 10 \right\}$$

Part three: solution

It is recomended to use a computer with some software that can create the graphs.

Notes

Create your own paintings!